Section 6 Continuous Random Variables

Statistics I · Fall 2026 · Zong-Wei Yeh · Department of Finance, CCU

Overview

Section 5 named the discrete distributions, and this section does the same for the continuous ones. A continuous model is defined by a density rather than a pmf, so probabilities become integrals and sums become areas. Everything else carries over. Each model is summarized by its mean and its variance. Where the integral is tractable, it also gets a moment generating function. The section builds to the most important distribution in statistics, the normal.

From a pmf to a pdf. Recall three facts from Section 4 about a continuous variable X with density f. First, a probability is an area under f: $P(a < X \le b) = \int_a^b f(x)\,dx$. Second, a single point has probability zero: $P(X = x) = 0$. So $<$ and $\le$ give the same probability. Third, the CDF and the density are linked by $F'(x) = f(x)$ wherever f is continuous. We integrate f to get F, and we differentiate F to get f.

This section has four parts.

- §6.1 covers the uniform, which spreads probability evenly over an interval. Every other model here is measured against it.
- §6.2 covers the exponential, the waiting time for a Poisson event. It is memoryless.
- §6.3 adds independent exponentials to get the Gamma. Section 7 builds the chi-square from it.
- §6.4 covers the normal. It gives the bell curve and the one table that every normal reduces to. It also gives the properties that make the normal easy to use.

Two pairings with Section 5 organize the material. The exponential is the continuous version of the geometric, and both are memoryless. The Gamma is a sum of independent exponentials. In the same way, the binomial is a sum of Bernoullis. We prove both facts by multiplying MGFs. The normal is *preserved* under sums of independent variables, which is why a portfolio of normal returns is itself normal. Section 7 builds directly on this section. Inference uses the normal, the Gamma, the chi-square and the Central Limit Theorem. Section 7 also shows that the normal approximates the binomial.

Distribution	pdf and support	Mean	Variance	MGF
$U(a,b)$				
$\text{Exp}(\lambda)$				
$\text{Gamma}(\alpha,\lambda)$				
$N(\mu,\sigma^2)$				

Fill in each row when you reach its subsection. Every entry is derived below, and §6.4 closes with a map of how the four are related. The uniform's MGF exists, but the course does not use it: its mean and variance come faster by direct integration.

6.1 The Uniform Distribution

Definition 6.1 (Uniform distribution)**.**
X is uniform (均匀分配) *on $[a,b]$, with $a<b$, written $X\sim U(a,b)$, if it has the constant density*

$$f(x)= \qquad ,\quad a\le x\le b, \qquad f(x)=0 \text{ otherwise.}$$

A uniform variable is equally likely to land anywhere in the interval. Probability is proportional to length. So this is the continuous version of "equally likely outcomes." Its distribution function is a straight ramp (Figure 1).

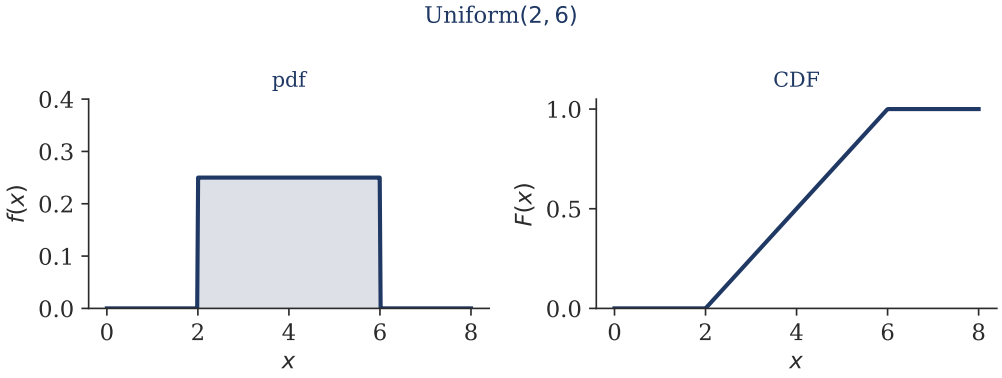


Figure 1: Uniform$(2,6)$. *Left:* the density is flat, so the probability of a subinterval is proportional to its length. *Right:* the CDF is a straight ramp across the interval.

Proposition 6.2 (Uniform CDF)**.**
For $a \leq x \leq b$, $F(x) =$ *(with* $F = 0$ *below* a, $F = 1$ *above* b*).*

Proof.

□

Proposition 6.3 (Uniform mean and variance)**.**
$\mathbb{E}(X) =$ *and* $\mathrm{Var}(X) =$.

Proof.

□

Example 6.4. $X \sim \mathrm{U}(0, 10)$. Find $P(X \leq 3)$, $P(2 < X < 7)$, $\mathbb{E}(X)$, and $\mathrm{Var}(X)$.

Example 6.5. A bus is equally likely to arrive at any time in the next 15 minutes. Find the probability you wait more than 10 minutes, and your expected wait.

6.2 The Exponential Distribution

Definition 6.6 (Exponential distribution)**.**
X *is* exponential (指數分配) *with rate* $\lambda > 0$*, written* $X \sim Exp(\lambda)$*, if it has density*

$$f(x) = \qquad , \qquad x \geq 0.$$

The exponential is the model for a waiting time. Suppose events arrive as a Poisson process at rate λ (Section 5). Then the wait until the next event is Exp(λ). Proposition 6.8 below proves this. A larger λ means shorter waits and a faster-decaying density (Figure 2).

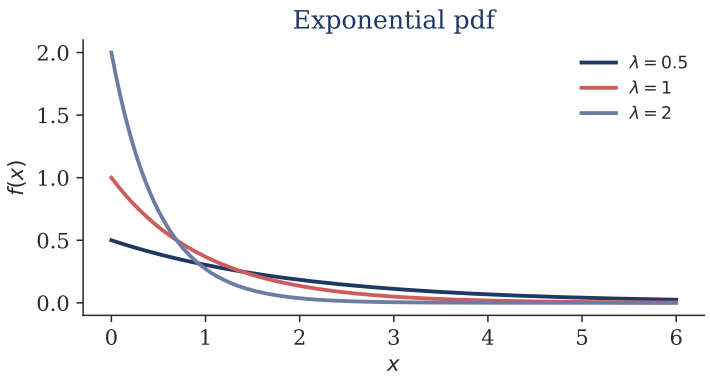


Figure 2: Exponential densities. Each one starts high and then falls away. A larger λ pushes the waiting time closer to 0, so events come sooner.

Proposition 6.7 (Exponential CDF and tail)**.**
$F(x) =$ *and* $P(X > x) =$ *for* $x \geq 0$.

Proof.

□

Proposition 6.8 (The wait in a Poisson process)**.**
Suppose that the number of events in any time window of length x is Poisson(λx). Let X be the wait from time 0 until the first event. Then $P(X > x) = e^{-\lambda x}$ for $x \geq 0$. So $X \sim Exp(\lambda)$.

Proof.

□

Section 5 stated the Poisson count for a window of one unit. This proposition assumes the same for a window of any length x, with mean count λx. The Poisson counts the events in a window. The exponential measures the time between events. Both describe the same process.

Proposition 6.9 (Exponential MGF, mean, variance)**.**
$M_X(t) =$ *for* $t < \lambda$*; hence* $\mathbb{E}(X) =$ *and* $\mathrm{Var}(X) =$.

Proof.

□

A higher rate means events come sooner. So the average gap between them is shorter.

Example 6.10. Events occur at rate $\lambda = 0.5$ per hour, so waits are $X \sim \text{Exp}(0.5)$. Find $P(X > 3)$, $\mathbb{E}(X)$, $\text{Var}(X)$, and $P(X < 1)$.

A problem may give the mean wait instead of the rate. Each is the reciprocal of the other. For example, a mean wait of 4 hours means $\lambda = 1/4 = 0.25$ per hour.

Theorem 6.11 (What the past wait tells us)**.**
For $s, t \geq 0$, $P(X > s + t \mid X > s) = P(X > t)$.

Proof.

□

An exponential variable is _____. Suppose you have already waited s hours. The wait that remains then has the *same* distribution as a fresh one. A used component is "as good as new." The geometric behaved the same way in Section 5. The exponential is the only continuous distribution with this property. The geometric is the only discrete one. We do not prove these uniqueness claims here.

Example 6.12. A component's lifetime is exponential with mean 1000 hours. Find $P(X > 1500)$, and use memorylessness to find $P(X > 1500 \mid X > 500)$.

The exponential is the continuous version of the geometric of Section 5. The table puts the two side by side. Fill in the first four rows of the exponential column.

	Geometric	**Exponential**
X measures	trials until the first success	
Support	$1, 2, 3, \ldots$	
$P(X > x)$	q^x, with $q = 1 - p$	
Mean	$1/p$	
$P(X > s + t \mid X > s)$	$P(X > t)$	$P(X > t)$

Example 6.13. Trades in a stock arrive as a Poisson process at 4 per minute. Let X be the wait in minutes until the next trade.

(a) Find $P(X > 0.5)$ in two ways: from the exponential tail, and from the number of trades in half a minute.

(b) Find the mean wait in seconds.

(c) Find the probability that the wait is under 15 seconds.

Remark (probability integral transform). Let X have a continuous, strictly increasing CDF F. Then $F(X)$ is uniform on $(0, 1)$. In the other direction, if U is uniform on $(0, 1)$, then $F^{-1}(U)$ has CDF F. We state this without proof. It is how a computer draws from a distribution: it draws U and returns $F^{-1}(U)$. For the exponential, solving $u = 1 - e^{-\lambda x}$ gives $F^{-1}(u) = -\ln(1-u)/\lambda$.

6.3 The Gamma Distribution

§6.2 gave the wait for one event. Now suppose a job has several stages. Each stage takes an independent exponential time. This subsection finds the distribution of the total time. The answer is the Gamma distribution. Its density uses one new function, so we define that first.

Definition 6.14 (Gamma function).
The Gamma function (伽瑪函數) *is*

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx, \qquad \alpha > 0.$$

The Gamma function is a number for each $\alpha > 0$. It is not a distribution. One value, which Section 7 will need, is $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. It follows from a result in §6.4. The proof of that result needs a double integral in polar coordinates, which is beyond this course. Every other value we need comes from the next result.

Proposition 6.15 (Recursion of the Gamma function).
For $\alpha > 0$, $\Gamma(\alpha + 1) =$. *So for a positive integer* n, $\Gamma(n) =$.

Proof.

□

The recursion holds for every $\alpha > 0$, not only for the integers.

Definition 6.16 (Gamma distribution).
X *is* Gamma (伽瑪分配) *with shape* (形狀參數) $\alpha > 0$ *and rate* $\lambda > 0$, *written* $X \sim \mathit{Gamma}(\alpha, \lambda)$, *if it has density*

$$f(x) = \qquad , \qquad x > 0.$$

The case $\alpha = 1$ *is the exponential.*

The Gamma distribution generalizes the exponential. As the shape α grows, the density moves away from 0 and becomes bell-like (Figure 3). The reason is the next two results. A Gamma is a sum of exponentials.

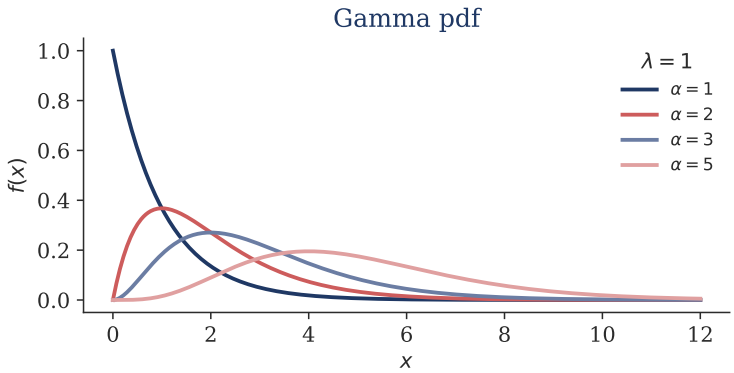


Figure 3: Gamma densities at rate $\lambda = 1$. At $\alpha = 1$ it is the exponential. As α increases the peak moves right and the shape becomes more symmetric. It already resembles the normal curve of §6.4.

Proposition 6.17 (Gamma MGF, mean, variance)**.**
$M_X(t) =$ *for* $t < \lambda$*; hence* $\mathbb{E}(X) =$ *and* $\mathrm{Var}(X) =$.

Proof.

□

Proposition 6.18 (Sum of independent exponentials)**.**
If $X_1, \ldots, X_n$ *are independent* $Exp(\lambda)$*, then* $\sum_{i=1}^{n} X_i \sim Gamma(n, \lambda)$*.*

Proof.

□

So the Gamma is to the exponential what the binomial is to the Bernoulli. In both cases we add independent copies of one simple variable, and in both cases the MGF gives the answer.

Example 6.19. $X \sim \text{Gamma}(2, 1)$, which is the sum of two independent Exp(1) waits. Find $\mathbb{E}(X)$, $\text{Var}(X)$, and $P(X \leq 2)$.

Proposition 6.20 (Sum of independent Gammas)**.**
If $X \sim \text{Gamma}(\alpha_1, \lambda)$ *and* $Y \sim \text{Gamma}(\alpha_2, \lambda)$ *are independent, with the* same λ*, then* $X + Y \sim$.

Proof.

□

This generalizes Proposition 6.18, which is the case of n shapes all equal to 1. Section 7 uses it constantly: the chi-square is a Gamma, so adding independent chi-squares adds their degrees of freedom.

Remark (chi-square bridge). The *chi-square* distribution with k degrees of freedom is exactly $\text{Gamma}(\alpha = k/2, \lambda = \frac{1}{2})$. It is the basis of the sampling distributions in Section 7. We introduce it there.

Remark (rate or scale). Some textbooks do not use the rate λ. They use $\beta = 1/\lambda$, which is called the *scale*. With the scale, the two densities are

$$\text{exponential: } f(x) = \frac{1}{\beta}e^{-x/\beta}, \qquad \text{Gamma: } f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-x/\beta}.$$

These are the same distributions. Only the parameter changes. With the scale, the exponential has mean β. The Gamma has mean $\alpha\beta$ and variance $\alpha\beta^2$. For example, rate $\lambda = 0.5$ is scale $\beta = 2$. These notes always use the rate. Another book or a software package may use the scale. Check which one it uses before you read Gamma$(\alpha, 2)$.

Example 6.21. A repair has three stages. Each stage takes an Exp(2) time in hours, independently of the others. Let X be the total repair time.

(a) Name the distribution of X. Find its mean, its variance and its standard deviation.

(b) A second repair has two such stages. It is independent of the first repair. Let Y be its total time. Name the distribution of $X + Y$, and find its mean and its variance.

6.4 The Normal Distribution

The normal is the bell curve, and it is the central distribution of statistics. The normal arises whenever many small independent effects add up. That is the Central Limit Theorem of Section 7. The theorem is why heights, measurement errors, and aggregate returns are so often close to normal.

Definition 6.22 (Normal distribution).
X *is* normal (常態分配) *with parameters* μ *and* $\sigma^2 > 0$, *written* $X \sim N(\mu, \sigma^2)$, *if it has density*

$$f(x) = \qquad\qquad\qquad , \qquad -\infty < x < \infty.$$

That this density integrates to 1 rests on the _____, $\int_{-\infty}^{\infty} e^{-z^2/2}\,dz = \sqrt{2\pi}$. Its proof needs a double integral in polar coordinates. That is beyond the single-variable calculus

this course assumes, so we take the result as known. The two parameters turn out to be exactly the mean and the variance, and Proposition 6.24 proves it. Changing μ slides the bell along the axis. Changing σ stretches it (Figure 4).

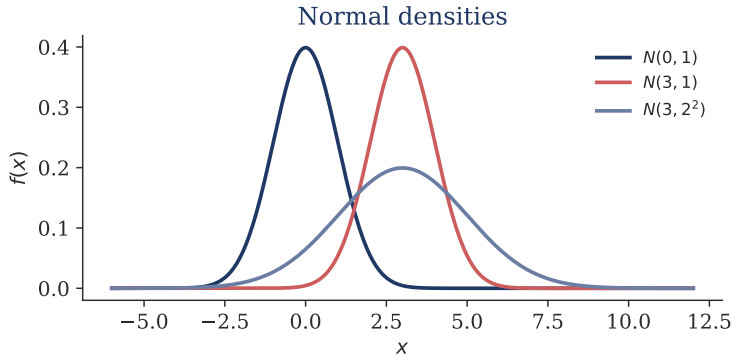


Figure 4: Normal densities. Shifting μ moves the bell. Enlarging σ flattens and widens it, and the area stays 1.

Proposition 6.23 (Shape of the normal density)**.**
Let $X \sim N(\mu, \sigma^2)$ with density f.

(i) $f(\mu - d) = f(\mu + d)$ *for every d. The density is symmetric about μ.*

(ii) $f(x) \leq f(\mu)$ *for every x. The density is highest at μ.*

(iii) $P(X \leq \mu) = 0.5$. *So μ is the median of X: half of the probability lies below it.*

Proof.

□

So the bell has its peak and its median at μ. Its left half and its right half have the same shape. Proposition 6.24 shows that μ is also the mean.

Proposition 6.24 (Normal MGF, mean, variance)**.**
$M_X(t) =$ *; hence* $\mathbb{E}(X) = \mu$ *and* $\mathrm{Var}(X) = \sigma^2$.

Proof.

□

Proposition 6.25 (A linear function of a normal)**.**
If $X \sim N(\mu, \sigma^2)$ *and* $a \neq 0$, b *are constants, then*

$$aX + b \sim N(a\mu + b, \quad).$$

Proof.

□

Section 4 already gave the mean and the variance of $aX + b$ for any X. The new fact is that the result is again normal. So a change of units, for example from percent to dollars, keeps a normal variable normal.

Definition 6.26 (Standard normal and the Φ table)**.**
$Z \sim N(0,1)$ *is the* standard normal (標準常態分配)*. Its density is* $\phi(z) =$, *and its CDF is* $\Phi(z) = P(Z \le z)$*, tabulated in the appendix. Symmetry of* ϕ *about* 0 *(Proposition 6.23) gives* $\Phi(-z) =$.

Proposition 6.27 (Reading the Φ table)**.**
Let $Z \sim N(0,1)$ *and let* $a < b$*.*

(i) $P(Z > a) =$.

(ii) $P(a < Z \le b) =$.

(iii) For $a > 0$, $P(-a < Z < a) = 2\Phi(a) - 1$.

Proof.

□

The table gives only left-tail areas $\Phi(z)$, and only for $z \ge 0$. These three rules and the symmetry rule of Definition 6.26 turn every probability about Z into table entries (Figure 5). Z is continuous, so $<$ and $\le$ can be exchanged freely.

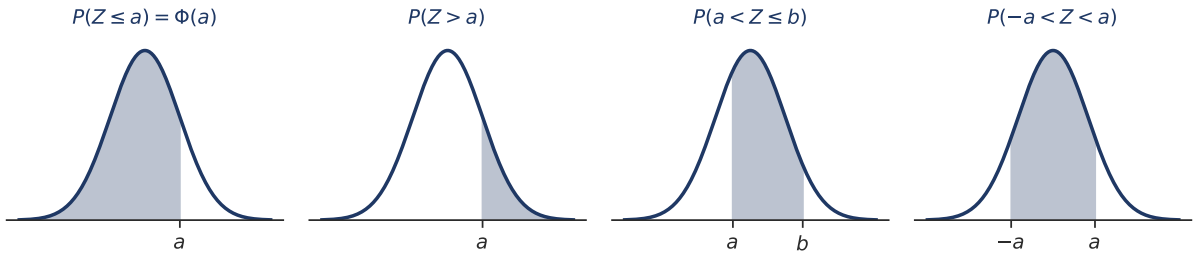


Figure 5: Four areas under the $N(0,1)$ density. The table gives the first one directly. Proposition 6.27 turns each of the other three into table entries.

Example 6.28. Let $Z \sim N(0,1)$. Use the table to find $P(Z \leq 1.25)$, $P(Z > 0.84)$, $P(-0.50 < Z \leq 1.50)$, and $P(-1.75 < Z < 1.75)$.

The _____ is the one curve we can actually look up. Every normal reduces to it. The recipe has a name: we _____ the variable. Proposition 6.29 states the recipe and proves that it works.

Proposition 6.29 (Reducing any normal to Z)**.**
If $X \sim N(\mu, \sigma^2)$ *then* $Z =$ $\sim N(0,1)$. *Hence* $P(X \leq x) =$.

Proof.

□

This is why one table is enough for the whole family. There are infinitely many normal distributions, one for each pair (μ, σ^2). Every question about any of them becomes a question about Z.

Example 6.30. $X \sim N(100, 15^2)$. Find $P(X < 115)$, $P(85 < X < 130)$, and $P(X > 118)$.

So far each question gave a value and asked for a probability. Some questions go the other way. They give a probability p and ask for the value x with $P(X \leq x) = p$. This value is the $100p$-th percentile of X. To find it, read the table backward. Look for p inside the table, and read z from the margins.

Example 6.31. (a) $X \sim N(100, 15^2)$. Find the 90th percentile and the 5th percentile. Then find the interval centered at 100 that contains X with probability 0.95.

(b) A machine fills bags of rice. The weight of a bag is $N(\mu, 5^2)$ in grams, and we can set μ. Find the μ for which only 1% of bags weigh less than 500 grams.

Example 6.32. A fund holds 5 million dollars. Its daily return is $R \sim N(0.1, 2^2)$, in percent. Its daily cost is 2 thousand dollars. Let Y be the profit of one day in thousand dollars.

(a) Write Y as a linear function of R, and find its distribution.

(b) Find the probability that the fund loses more than 100 thousand dollars in a day.

Theorem 6.33 (Linear combinations of independent normals)**.**
If $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$ are independent, then for constants a, b, not both zero,

$$aX + bY \sim N\big(a\mu_X + b\mu_Y, \qquad\qquad\big).$$

Proof.

□

So a portfolio of independent normal returns is *itself* normal. Section 4 gave us its mean and its variance. Now we have its whole distribution. That is what we need to price risk or to compute the chance of a loss.

Example 6.34. Independent returns (in %) are $R_1 \sim N(8, 10^2)$ and $R_2 \sim N(12, 20^2)$. A portfolio holds 60% in asset 1 and 40% in asset 2. Find the portfolio return's distribution and $P(R_p < 0)$.

The section has four models. To choose one, ask what X measures.

X **is**	**Model**
equally likely to be anywhere on an interval	Uniform
the wait until the next event, at a constant rate	Exponential
the total of n independent Exp(λ) waits	Gamma
a total of many small independent effects	Normal

Figure 6 collects the section. The two arrows into the dashed box belong to Section 7. It defines the chi-square as this Gamma, and it proves that the square of a standard normal is a chi-square. The arrow from the uniform is the Remark in §6.2, which we stated without proof. Every other arrow was proved above.

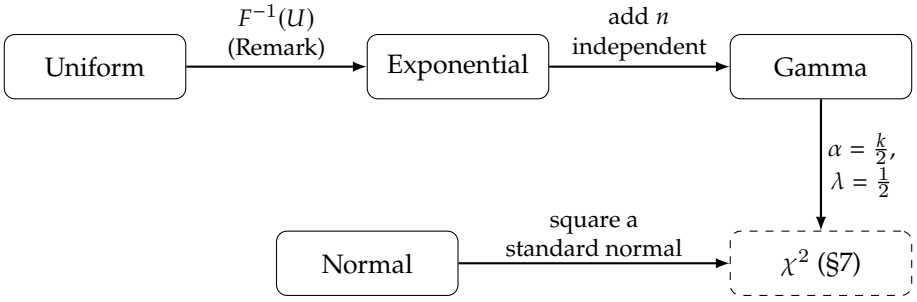


Figure 6: The continuous models of this section. Inverting a CDF turns a uniform into any of them, for example the exponential. Adding independent exponentials gives the Gamma. The Gamma with shape $k/2$ and rate $\frac{1}{2}$ is the chi-square. Section 7 shows that the square of a standard normal is a chi-square with one degree of freedom. That is how the normal and the Gamma meet.

Exercises

Exercise 6.1. *$X \sim U(2, 8)$. Find $\mathbb{E}(X)$, $\mathrm{Var}(X)$, $P(X < 5)$, and $P(X > 6)$.*

Exercise 6.2. *A taxi arrives uniformly over the next 12 minutes. Find the probability the wait is between 3 and 8 minutes, and the expected wait.*

Exercise 6.3. *$X \sim U(a, b)$ has $\mathbb{E}(X) = 10$ and $\mathrm{Var}(X) = 12$. Find a and b, and then $P(X > 13)$.*

Exercise 6.4. *Tomorrow's stock price is $X \sim U(90, 110)$. A call option pays $\max(X - 100,\, 0)$. Find the probability that the option pays anything, and its expected payoff.*

Exercise 6.5. *X is exponential with mean 4 (so $\lambda = 0.25$). Find $P(X > 6)$ and $\mathrm{Var}(X)$.*

Exercise 6.6. *For the exponential of Exercise 6.5 (mean 4), use memorylessness to find $P(X > 10 \mid X > 4)$.*

Exercise 6.7. *A light bulb's lifetime is exponential with mean 800 hours. Find the median lifetime. Then find the probability that the bulb fails before its mean lifetime.*

Exercise 6.8. *$X \sim Exp(\lambda)$ and $P(X > 2) = 0.25$. Find λ and $\mathbb{E}(X)$. Then find $P(X > 4 \mid X > 2)$ without any new integral.*

Exercise 6.9. *$X_1, \dots, X_4$ are independent $Exp(1)$. Name the distribution of $\sum_{i=1}^{4} X_i$ and give its mean and variance.*

Exercise 6.10. *A job has two stages. Each stage takes an exponential time with mean 2 hours, independently of the other. Name the distribution of the total time X, and find $P(X > 6)$.*

Exercise 6.11. *Use the recursion of Proposition 6.15 to find $\Gamma(5)$ and $\Gamma(\frac{7}{2})$.*

Exercise 6.12. *$X \sim Gamma(\alpha, \lambda)$ has mean 6 and variance 12. Find α and λ. Then write X as a sum of independent exponentials.*

Exercise 6.13. *$Z \sim N(0, 1)$. Find $P(Z < 1.50)$, $P(Z > 2.00)$, and $P(-1 < Z < 1)$.*

Exercise 6.14. *$Z \sim N(0, 1)$. Find the number c with $P(-c < Z < c) = 0.95$.*

Exercise 6.15. *$X \sim N(50, 10^2)$. Find $P(X < 65)$, $P(X > 35)$, and the 95th percentile.*

Exercise 6.16. *Exam scores are $N(500, 100^2)$. Find the cutoff for the top 10%.*

Exercise 6.17. *Independent returns are $R_1 \sim N(6, 6^2)$ and $R_2 \sim N(10, 8^2)$. An equal-weighted portfolio is formed. Find its distribution and $P(R_p < 0)$.*

Exercise 6.18. *Independent $X \sim N(3, 9)$ and $Y \sim N(1, 16)$. Name the distribution of $X + Y$ and find $P(X + Y > 9)$.*

Exercise 6.19. *A fund's daily return, in percent, is $N(0.05,\, 1.2^2)$. Find the return x with $P(R < x) = 0.05$. Risk managers call $-x$ the 5% value at risk.*

Exercise 6.20. *A machine fills bottles with $X \sim N(500, 4^2)$ ml. The bottles hold $Y \sim N(510, 3^2)$ ml, independently of the fill. A bottle overflows when $X > Y$. Find the probability of an overflow.*

Exercise 6.21. *A broker's system processes each order after a delay of X seconds, where $X \sim U(0, 6)$. The delays of different orders are independent. An order is* slow *if its delay is more than 4.5 seconds.*

(a) Find the probability that an order is slow.

(b) A client sends 8 orders. Find the probability that at most one of them is slow.

(c) The client keeps sending orders. Find the expected number of orders up to and including the first slow one.

(d) An order has already waited 3 seconds and is still not processed. Find the probability that it will be slow. Is the uniform memoryless?

Exercise 6.22. *A phone's lifetime X, in years, is exponential with mean 5. The maker gives a warranty of w years. If the phone fails within w years, the maker pays a claim.*

(a) Let $w = 1$. Find the probability of a claim.

(b) The maker wants at most 10% of phones to lead to a claim. Find the largest warranty w that allows this.

(c) The maker earns 100 dollars on each phone sold. A claim costs the maker 400 dollars. Find the expected profit per phone when $w = 1$. Then find the largest w for which the expected profit is not negative.

(d) A phone still works at the end of a 2-year warranty. Find the probability that it works for at least 3 more years.

Exercise 6.23. *A bank has two trading desks. On any day, desk A earns $X \sim N(2, 3^2)$ and desk B earns $Y \sim N(1, 4^2)$, both in million dollars. X and Y are independent. Let $T = X + Y$ be the bank's total profit for the day.*

(a) Find the distribution of T.

(b) Find the probability that the bank loses money on a day.

(c) The bank holds capital of c million dollars. It wants the probability that a day's loss is more than c to be at most 0.01. Find the smallest such c.

(d) Different days are independent. Find the probability that the bank loses money on at least one day of a 5-day week.

Exercise 6.24. *Among fund managers, 20% are skilled and 80% are not. A skilled manager's yearly return, in percent, is $N(8, 10^2)$. An unskilled manager's yearly return is $N(2, 10^2)$. We pick one manager at random and look at one year's return R.*

(a) For each type of manager, find the probability that $R > 12$.

(b) Find $P(R > 12)$ for the manager we picked.

(c) The manager's return is above 12. Find the probability that the manager is skilled.

(d) Repeat (c) for a return above 22. Which return is stronger evidence of skill? In each case, is the manager more likely skilled or unskilled?

Exercise 6.25. *A consultant's job takes T hours, where T is exponential with mean 2. She charges 1000 dollars for each hour billed. Her minimum bill is one hour. So the billed hours are $B = 1$ when $T \leq 1$, and $B = T$ when $T > 1$.*

(a) Find the probability that the job takes at most one hour.

(b) Suppose the job takes more than one hour. Use Theorem 6.11 to find the distribution of the extra time $T - 1$. Give its mean.

(c) Find $\mathbb{E}(B)$ and the expected fee. No integral is needed.

(d) A client asks for a cap of 4 billed hours. So the billed hours become $C = B$ when $T \leq 4$, and $C = 4$ when $T > 4$. Find $\mathbb{E}(C)$.

Appendix: Standard Normal Table

Cumulative probabilities $\Phi(z) = P(Z \leq z)$ for the standard normal. Read the row for the first two digits of z and the column for the second decimal. For example, $\Phi(1.24)$ is row 1.2, column $.04$. For negative z use $\Phi(-z) = 1 - \Phi(z)$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998